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Baryons from supergravity

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Abstract: We study the construction of baryons via supergravity along the line suggested recently by Witten and by Gross and Ooguri. We calculate the energy of the baryon as a function of its size. As expected the energy is linear with N. For the non-supersymmetric theories (in three and four dimensions) we find a linear relation which is an indication of confinement. For the $\mathcal{N}=4$ theory we obtain the result ($EL = -$const.) which is compatible with conformal invariance. Surprisingly, our calculation suggests that there is a bound state of k quarks if $N \geq k \geq 5N/8$. We study the $\mathcal{N}=4$ theory also at finite temperature and find the zero temperature behavior for small size of the baryon, and screening behavior for baryon, whose size is large compared to the thermal wavelength.

Keywords: 1/N Expansion, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Recently, [1] it was conjectured that four-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$ is dual to type IIB string theory on the background $AdS_5 \times S^5$ (where AdS is five-dimensional anti-de-Sitter space). In this correspondence the string coupling g_s is equal to the gauge coupling g^2_{YM} and $(g^2_{YM}N)^{1/4} \equiv (g^2_{eff})^{1/4}$ is proportional to the radius of the AdS space and the five-sphere (in string units). There are N units of five-form flux on the S^5 in string theory. In the limit of large N and large g^2_{eff} the string theory is reliably approximated by supergravity and one expects to be able to extract gauge theory correlations functions, the set of chiral operators and the mass-spectrum of the strongly coupled gauge theory using classical calculations in supergravity.

A precise relationship between the supergravity effective action and gauge theory correlators [2, 3] and a match of chiral primary operators in the conformal theory with Kaluza-Klein states of the compactified supergravity [3] has been found. Likewise, there exist precise recipes for computing the Wilson loop operators [5, 6]. This allows us to study the qualitative behavior of gauge theories (confinement, screening, ...) at zero or finite temperature and also of non-supersymmetric theories [4, 11, 13]. The relevant configuration of external quark and anti-quark can be thought of as a mesonic vertex operators. In the supergravity description they are constructed as an open fundamental string connecting to separated points on the boundary of AdS space where the string endpoints correspond to the external quark and anti-quark, respectively.

More recently the construction of baryons was discussed in the supergravity framework [7, 8]. The precise meaning of a baryon in this context is a finite energy configuration of N external quarks. The $\mathcal{N} = 4$ SYM theory does not contain dynamical quarks in the fundamental representation which are necessary to construct baryonic particles. (An interesting counter example to this is the "Pfaffian" particle which is constructed out of adjoint fields in $\mathcal{N}=4$ with $SO(2N)$ gauge group [7]).

In the construction of a baryon vertex in string theory one faces a puzzle. If we think of the N quarks as endpoints on the AdS boundary of N fundamental strings with equal orientation it seems a priori inconsistent to let the other ends of the strings terminate on one point in the interior of AdS. Nevertheless, it was shown in [7] that this is possible (see [8] for a different argument involving the Chern-Simons term of the compactified supergravity). The baryon vertex turns out to be a D5 brane wrapped on the S^5. In the type IIB string theory there is a self-dual field strength G_5 and, as mentioned earlier, the compactification on $AdS_5 \times S^5$ has N units of flux on the five-sphere: $\int_{S^5} \frac{G_5}{2\pi} = N$. On the D5 brane world volume there is a $U(1)$ gauge field A which couples to the five-form field strength through the term $\int_{R\times S^5} A \wedge \frac{G_5}{2\pi}$. Because of this coupling G_5 contributes N units of $U(1)$ charge. Each string endpoint adds -1 unit of charge Since in a compact space the total charge has to vanish, precisely N strings have to end on the D5 brane. In the $SU(N)$ gauge theory the gauge invariant combination of N quarks is completely antisymmetric and, indeed, the strings between the boundary (or a D3 brane) and the D5 brane are fermionic strings [7] because the strings have mixed DN boundary conditions in eight space directions.

In the supergravity description of non-conformal theories [9] we can use similar arguments. The starting point is a set of N Dp-branes which give rise to N units of flux of a $p+2$-form field strength G_{p+2} of the type II string theory: $\frac{1}{2\pi}\int_{S^{8-p}} \star G_{p+2} = N$. The baryonic vertex is represented by a D$(8-p)$-brane wrapped on an S^{8-p} with U dependent radius. The $U(1)$ gauge field A on the D$(8-p)$-brane couples to G_{p+2} through the term $\frac{1}{2\pi}\int_{R\times S^{8-p}} A \wedge \star G_{p+2}$ and, therefore, leads to N units of $U(1)$ charge which are canceled by $-N$ units of charge from N fundamental strings ending on the D$(8-p)$-brane.

In the present letter we want to study baryonic vertices in detail and calculate the energies of such configurations in string theory. The energy has two main contributions, the tension of the strings and the energy of the D5 brane (we will not discuss corrections due to interactions between the strings). Both contributions are proportional to N but come with opposite signs. Therefore, stable configurations exist and we calculate the energy as a function of the characteristic size L of the baryon. For a static configuration we have to demand that the net force on the vertex vanishes. Using this constraint we determine the angle at which the strings end on the vertex. We study baryon in four dimensional theories with maximal supersymmetry at zero and finite temperature and non-supersymmetric theories in three and four dimensions and compare the results to Wilson loop calculations. Furthermore, our calculation shows that there is a bound state of k quarks if k satisfies $N \geq k \geq 5N/8$. The expression for the baryon energy in maximally supersymmetric theories of other dimensions is also written down.

2. Baryons of $\mathcal{N} = 4$ SYM in four dimensions

Consider the baryon configuration suggested in [7]. There are two contributions to the action of the system. The first contribution comes from the string stretched between the boundary of the AdS$_5$ space and the D5-brane wrapped on the S_5. The second contribution comes from the D5-branes itself. As was noted in [7] they are of the same order. Hence, we should consider both of them. Let us start with the D5-brane. Since we are considering a static D5-brane wrapped on S^5 its action is

$$S_{D5} = \frac{1}{(2\pi)^5 {\alpha'}^3 e^{\phi}} \int dx^6 \sqrt{h} = \frac{TNU_0}{8\pi}\,, \tag{2.1}$$

where U_0 is the location of the baryon vertex in the bulk, T is the time period which we take to infinity and h is the induced metric on the fivebrane.

The configuration which we consider (see Fig. 1) is such that the strings end on a surface with radius L in a symmetric way which ensures that the net force on the vertex along x_i vanishes (where x_i are the direction along the boundary where the field theory is living). Hence the configuration is stable in the x_i directions. Of course to stabilize the system along the U direction the symmetry argument is not enough, one has to consider the strings action as well.

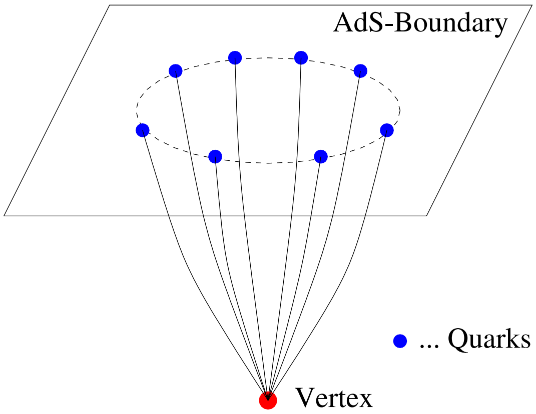

Figure 1: The baryon vertex.

Following [6], we work with the Nambu-Goto action in the gauge $x = \sigma$ and $t = \tau$ which gives

$$S_{1F} = \frac{T}{2\pi} \int dx \sqrt{U_x^2 + U^4/R^4}\,, \tag{2.2}$$

where $U_x = \frac{\partial U}{\partial x}$ and $R^4 = 4\pi g_s N$. The total action is

$$S_{total} = S_{D5} + NS_{1F}\,. \tag{2.3}$$

The variation of (2.3) under $U \to U + \delta U$ contains a volume term as well as a surface term. The volume term leads to the Euler-Lagrange equation whose solution satisfies [6]

$$\frac{U^4}{\sqrt{U_x^2 + U^4/R^4}} = \text{const.} \tag{2.4}$$

because the action does not depend explicitly on x.

The surface term yields

$$\delta U \frac{TN}{8\pi} = \delta U \frac{TN(U_x)_0}{2\pi \sqrt{(U_x)_0^2 + U_0^4/R^4}}\,, \tag{2.5}$$

where $(U_x)_0 = \frac{\partial U}{\partial x}|_{U_0}$ and δU is the variation of U at $x = 0$ where the string hits the baryon vertex. This condition is simply the no-force condition in the curved space-time. Using (2.4) and (2.5) one finds that

$$\frac{U^4}{\sqrt{U_x^2 + U^4/R^4}} = \sqrt{\frac{15}{16}} U_0^2 R^2 . \tag{2.6}$$

This implies the following relation between U_0 and the radius of the baryon L

$$L = \frac{R^2}{U_0} \int_1^\infty \frac{dy}{y^2 \sqrt{(\beta^2 y^4 - 1)}} , \tag{2.7}$$

where $\beta = \sqrt{16/15}$. The energy of a single string is given by

$$E = \frac{1}{2\pi} U_0 \left(\int_1^\infty dy \frac{\beta y^2}{\sqrt{\beta^2 y^4 - 1}} - 1 \right) - \frac{U_0}{2\pi} , \tag{2.8}$$

where we subtract the energy of the configuration with the D5-brane located at $U = 0$. Since g_{xx} vanishes at $U = 0$ any radial string which reaches this point ends on the D5-brane. As a result the energy of the fermionic strings, which we subtract equals the energy of free quarks.[1] Note that since $g_{tt}(U = 0) = 0$ the contribution of the D5-brane located at $U = 0$ to the energy vanishes.

Inserting the relation (2.7) into (2.8) one finds that the energy of each string is

$$E = -\alpha_{st} \frac{\sqrt{2 g_{YM}^2 N}}{L} , \tag{2.9}$$

where

$$\alpha_{st} = \frac{1}{4} \sqrt{\frac{5}{6\pi}} {}_2F_1[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}; \frac{15}{16}] \times {}_2F_1[\frac{1}{2}, -\frac{1}{4}, \frac{3}{4}; \frac{15}{16}] \simeq 0.036 .$$

The total energy of the baryon configuration is therefore

$$E = -\alpha_B N \frac{\sqrt{2 g_{YM}^2 N}}{L} , \quad \text{where} \quad \alpha_B = ... \simeq 0.007 . \tag{2.10}$$

Since the force $F = \frac{dE}{dL}$ is positive the baryon configuration is stable. Moreover, as expected from the field theory large N analysis, the energy is proportional to N times that of the quark anti-quark system. Recall [6] that while the fact that the energy is proportional to $1/L$ is dictated by conformal invariance, the dependence on R^2 is a non-trivial prediction of the AdS formulation concerning the strong coupling behavior of the gauge theory.

[1]By free quarks we mean string which are stretched from $U = \infty$ to $U = 0$

2.1. $\mathcal{N} = 4$ at finite temperature

In [11, 12] the Wilson loop at finite temperature was considered. The temperature introduces a scale into the conformal theory, which distinguish between the behavior in two regions. So there are two regions. At distances smaller then the wavelength associated with the temperature the behavior is essentially the conformal one ($E \sim -1/L$) with corrections. However, at large distances the charges are screened by the the effects of the temperature. From the supergravity point of view what is happening is the following. In the $T = 0$ case to increase L one has to decrease U_0 (the point where the slope , U_x, is zero). In the presence of a temperature one cannot decrease U_0 below the horizon, associated with the temperature, and hence it seems that we have a maximal distance of separation between the quark and the anti-quark. However, before we reach that point the energy becomes positive (after the subtraction of the free quarks energy) and hence the quark anti-quark system becomes free.

The situation with the baryons is rather similar although there are some technical differences. The surface term now yields,

$$\frac{U_0'}{\sqrt{(U_0')^2 + (U_0^4 - U_T^4)/R^4}} = \frac{1}{4}\left(\frac{1 + U_T^4/U_0^4}{\sqrt{1 - U_T^4/U_0^4}}\right). \tag{2.11}$$

We see that the minimal value of U_0 is not the location of the horizon U_t but γU_0 where $\gamma > 1$. The integral for the size L baryon takes the following form:

$$L = \frac{R^2}{U_0}\int_1^\infty dy \sqrt{\frac{15 - 18\rho^4 - \rho^8}{(y^4 - \rho^4)(16y^4 - 15 + 2\rho^4 + \rho^8)}}, \tag{2.12}$$

where $\rho = U_T/U_0$. Since the minimal U_0 is larger then in the Wilson loop case the maximal L is smaller than in the Wilson loop case. Thus one might worry that we reach L_{max} before we reach the positive energy condition. However, now the energy of the system contains also a positive term coming from the D5-brane[2]

$$E = \frac{NU_0}{2\pi}\left\{\int_1^\infty dy\left(\sqrt{\frac{y^4 - \rho^4}{16y^4 - 15 + 2\rho^4 + \rho^8}} - 1\right) - 1 + \rho + \frac{1}{4}\sqrt{1 - \rho^4}\right\}. \tag{2.13}$$

We therefore, reach the positive energy condition before we reach L_{max}.

It should be emphasized that the configuration where the D5-brane is at the horizon is static only from the field theory point of view. Namely, unlike the configuration where the D5-brane is above the horizon which is static because the net force at the vertex (including the gravitational one) is zero, here the net force on the vertex is positive. Therefore, the D5-brane falls into the black hole. However, from the point of view of an observer located at the boundary (a field theory observer) it takes the D5-brane an infinite amount of time to cross the horizon hence the configuration is static. We

[2]For an explanation on the subtraction see, Fig. 2.

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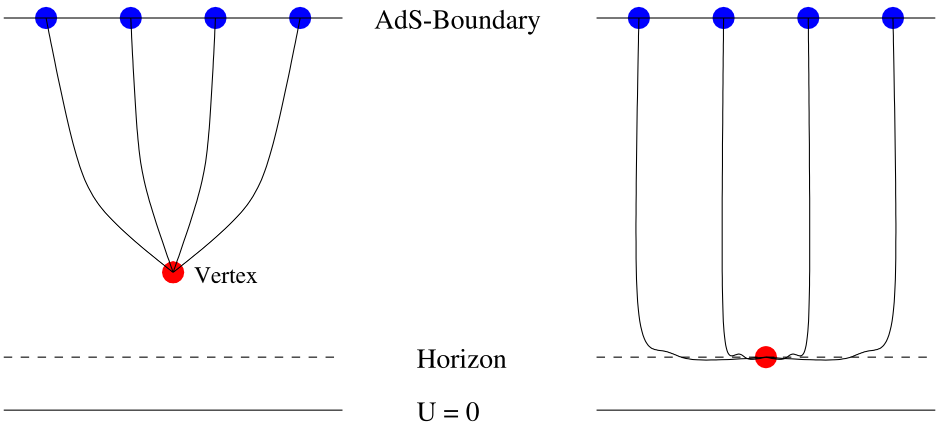

Figure 2: The two ways to obtains N fermionic quarks at the boundary. At small distances (compared to the wavelength of the temperature) the lowest energy configuration is the one with the D5-brane above the horizon. In this configuration there is a potential between the quarks and the vertex. At large distances the lowest energy configuration is the one with the D5-brane located at the horizon. Since g_{tt} vanishes at the horizon the contribution to the energy from the horizontal parts of the strings (as well as from the D5-brane) vanishes. Hence, the energy does not depend on the size of the baryon. In other words the energy of this configuration is simply N times the energy of a string stretched between $U = \infty$ and the horizon which explains the subtraction made at eq.(2.13).

should also note that since the D5-brane is a freely falling object it will not be burned by the uge Hawking temperature at the horizon.

To summarize the behavior of the total energy as a function of the size L is similar to the case of the quark-anti-quark pair. For small size we find a Coulomb like behavior but at a certain critical size L_c the energy becomes zero and we find that the baryon should decay into a configuration of N quarks with vanishing interaction.

2.2. Baryons in non-conformal field theories with sixteen supercharges

As was explained in the introduction one can generalize the supergravity construction of the baryons to the non-superconformal theories living on a collection of N Dp-branes (for $p \neq 3$) using the relevant supergravity solution [9]. The relation between the size of the baryons and the energy is basically N times the quark anti-quark potential found in [6, 11, 12, 13] (for $p \neq 5$),

$$E_{bar} \sim -N \left(\frac{g_{YM}^2 N}{L^2} \right)^{1/(5-p)} . \tag{2.14}$$

3. Baryons in non-SUSY theories

We discuss YM in three dimensions (the generalization to the four dimensional case is straight forward). The results which we obtain are expected from the field theory point of view and were anticipated in [8]. The supergravity solution associated with pure YM in three dimensions is given by the near-extremal D3-branes solution in the decoupling limit

$$ds^2 = \frac{U^2}{R^2}\left[-(1-\frac{U_T^4}{U^4})dt^2 + \sum_{i=1}^{3} dx_i^2\right] + \frac{R^2}{U^2(1-\frac{U_T^4}{U^4})}dU^2 + R^2 d\Omega_5^2\,. \tag{3.1}$$

To obtain three dimensional theory we need to go to the IR limit and to consider distances (along x_1, x_2, x_3) which are much larger then $1/T$. At the region where we can trust the supergravity solution, $R^2 \geq 1$, the theory is not quite three dimensional because the QCD string can probe the compactified direction [4, 13]. Nevertheless this theory possesses some properties of YM in three dimensions [4, 13, 14, 15].

The surface term gives

$$\frac{1}{4} = \frac{U_0'}{(1-U_T^4/U_0^4)\sqrt{U^4/R^4 + (U_0')^2/(1-U_T^4/U_0^4)}}\,. \tag{3.2}$$

To go to the IR limit we need to consider large L. As in the Wilson loop case this means that $U_0 \to U_T$. At this limit the integrals of L and E are controlled by the region near U_0 and their ratio is a constant which determined the QCD string tension. At first sight it seems that eq.(3.2) will change dramatically the relation between E and L. However, at the IR limit ($U_0 \to U_t$) eq.(3.2) implies that U_0' vanishes so the relation is again, as expected, linear

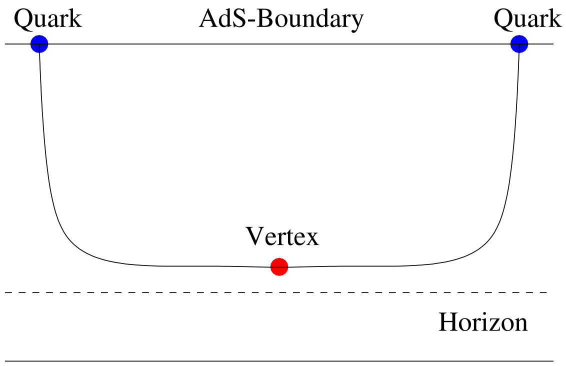

Figure 3: The baryon in non supersymmetric theories

$$E = NT_{YM}L\,, \quad \text{where} \quad T_{YM} = \pi R^2 T^2\,. \tag{3.3}$$

We should note the same relation holds for non-supersymmetric YM in four dimensions with the string tension found in [13].

4. Baryons with $k < N$ quarks

Next we would like to study baryons made of k quarks when $k < N$. For example the case $k = N-1$ gives rise to a baryonic configuration in the anti-fundamenatl representation. In a confining theory we do not expect to find such a state (it cost an infinite amount of energy to separate $N-k$ quarks all the way to infinity leaving behind the k-baryonic system). Indeed, as we shall see, in the non-supersymmetic theories this $k < N$ baryon configuration is excluded.Surprisingly in the $\mathcal{N} = 4$ theory we do find such stable k-quarks baryon if $5N/8 < k \leq N$. This is unexpected result which we do not really understand from the field theory point of view.

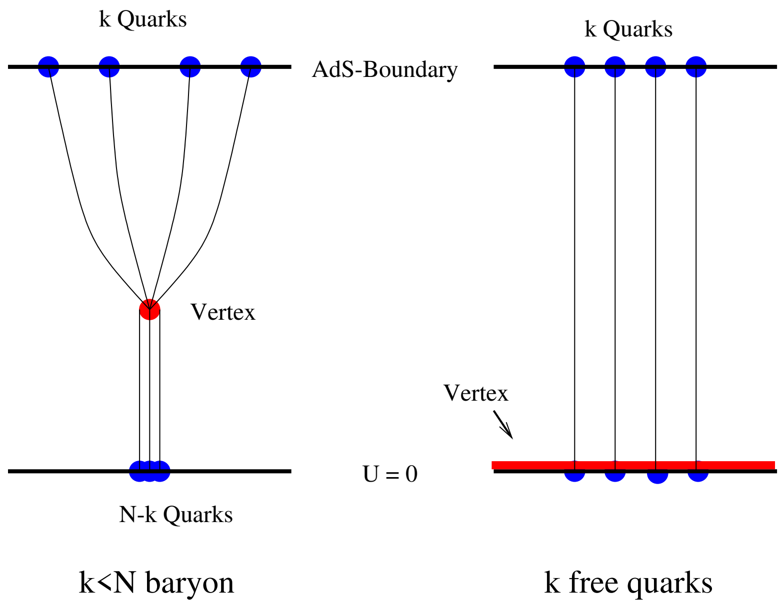

Figure 4: The $k < N$ "baryon" vs. k free quarks. Since the longitudinal metric vanishes at $U = 0$ the the surface $U = 0$ is in fact a point and hence the vertex is smeared along this "surface" $U = 0$. As a result the string can move freely at the boundary.

The way supergravity enables us to construct baryons with less quarks is illustrated in Fig. 4. In this figure we have the usual baryonic vertex with k strings stretched out to the boundary at $U = \infty$ and the rest $N - k$ strings reaching $U = 0$.[3]

This configuration is stable provided that $\frac{dE}{dL} > 0$. The calculation of the energy of this configuration proceeds in a similar way to the calculation of the energy of $k = N$ baryonic system carries in section 2. the surface term gives now the following relation:

$$\frac{U_x}{\sqrt{U_x^2 + U^4/R^4}} = A \,, \tag{4.1}$$

where

$$A = \frac{5N - 4k}{4k} \,.$$

For $k = N$ we get $A = 1/4$ and for $k < N$ we have $A > 1/4$. It follows from (4.1) that $A \leq 1$. The upper bound , $A = 1$ corresponds to $U_x \to \infty$ and $k = 5N/8$. since the strings are radial the baryon size vanishes.

The energy of the k-quarks baryon is

$$E_k = \frac{U_0}{2\pi}\left[(N-k) + N/4 + k\left(\int_1^\infty dy(\frac{y^2}{\sqrt{y^4 - (1 - A^2)}} - 1) - 1\right)\right] . \tag{4.2}$$

Where we have made the same kind of subtraction as in the $k = N$ configuration i.e. we have substracted the energy of k quarks as depicted in Fig. 4b. For $A = 1$ ($k = 5N/8$)

[3] Configurations with strings ending on $U = 0$ were also considered in the context of quark monopole potential [17]

the energy vanishes which implies that the location of the D5-brane is a moduli of the system. For $A < 1$ the energy is bU_0 with some negative b and U_0 is determined, as usual, in terms of L.

The fact that no k-quarks baryons exist once $k < 5N/8$ can be deduced by considering the surface relation at the D5-brane. It is easy to see that in this case not all the $N - k$ strings can go radially directed towards $U = 0$. Instead they should come out of the vertex with some finite slope and will therefore, never reach $U = 0$. Instead they will eventually end on the $U = \infty$ boundary leaving us with more quarks on this boundary.

We would like to end with a short remark on the non-supersymmetric case. As we remarked at the begging of this section, in a confining field theory we do not expect to find such states. This expectation seems to be supported by the AdS supergravity approach. The energy of a radial string is

$$E = \frac{1}{2\pi}\int_{U_0}^{U_1} dU \sqrt{G_{xx}G_{uu}} \sim \log(U_0 - U_T)\,. \tag{4.3}$$

Therefore, the energy of a string stretched between the D5-brane and the horizon is infinite and hence even the case $k = N - 1$ cost an infinite amount of energy. Thus the baryonic configuration with $k = N$ is the only stable baryonic configuration with finite energy in agreement with field theory results.

Acknowledgments

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